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Dynamic Behavior of Dissymmetric Rotor Bearings Modelled With a Periodic Coefficient Large System

P. M. Guilhen
P. Berthier
G. Ferraris
M. Lalanne

This work is concerned with the instability and unbalance response of dissymmetric rotor-bearing systems containing periodic coefficients when modeling produces matrices with a large number of degrees of freedom. It is important to solve the equations and then to predict the dynamic behavior of the system. This can be done knowing the instability areas, and the unbalance response in the stable areas. One deals here with a large number of equations and a reduction of the number of degrees of freedom of the system is achieved through a pseudo modal method. This method is shown to give satisfactory results.

INTRODUCTION

It is now more and more necessary to predict accurately the dynamic behavior of rotor bearings systems : natural frequencies as a function of the speed of rotation which gives the critical speeds and instability zones and response to unbalance and nonsynchronous forces.

For most of rotors containing a large number of degrees of freedom, calculations do not pose any problem today. Many authors, as [1], [2], [3], [4], have shown that, using a method based either on substructuring or on a modal reduction, the number of degrees of freedom can be lowered significately and the results are without significant loss of accuracy.

For dissymmetric rotors, it happens sometimes that the equations lead to periodic coefficients. In this case, some authors, as [5], [6], [7], [8] give methods to detect instability zones (mainly transfer-matrix or infinite determinant methods) and to calculate step by step the unbalance response. But usually, these methods are applied on systems having a few degrees of freedom (up to 20). The purpose of this study is to develop for large periodic systems, a method which can be used to obtain the dynamical behavior of the systems. In a first part, a pseudo-modal method adapted to the resolution of periodic differential equations is presented. In a second part an industrial application containing 96 degrees of freedom is considered. The influence of the number of modes on the results, and the accuracy of the methods are discussed.

PSEUDO-MODAL METHOD

The systeme to be solved can be written as :

$$MX^{\bullet\bullet} + A(t)\,X^{\bullet} + K(t)X = F(t) \qquad (1)$$

M, A, K are n order matrices and it has been shown [8] that A(t) and K(t) can be written as :

$$A(t) = A_o + A_1 \sin 2\Omega t + A_2 \cos 2\Omega t \tag{2}$$

$$K(t) = K_o + K_1 \sin 2\Omega t + K_2 \cos 2\Omega t \tag{3}$$

The pseudo-modal method consists in using a "modal" base of the system

$$M X^{\bullet\bullet} + \bar{K} X = 0 \tag{4}$$

The modal base is calculated from the system at rest ($\Omega = 0$) and at the initial instant ($t = 0$). In these conditions, there are neither periodic coefficients, nor gyroscopic effects. Furthemore, the damping of the bearings is omited, and the non symmetric terms of the stiffness matrix K are symmetrised :

$$K = K_o + K_2 \tag{5}$$

$$\bar{K} = \text{sym}(K) \tag{6}$$

The modal base ϕ is built with the lowest ℓ modes of (4), ($\ell << n$). The relation between the degrees of freedom of the system and the modal parameters is :

$$X = \phi q \tag{7}$$

Equation (1) with (2), (3) and (7) leads to :

$$\begin{aligned} M \phi q^{\bullet\bullet} + (A_o + A_1 \sin 2\Omega t + A_2 \sin 2\Omega t) \phi q^{\bullet} + (K_o + K_1 \sin 2\Omega t \\ + K_2 \cos 2\Omega t) \phi q = F \end{aligned} \tag{8}$$

and premultiplying by ϕ^t, (8) can be written as :

$$m q^{\bullet\bullet} + (a_o + a_1 \sin 2\Omega t + a_2 \cos 2\Omega t) q^{\bullet} + (k_o + k_1 \sin 2\Omega t + k_2 \cos 2\Omega t) q = f \tag{9}$$

with

$$\begin{aligned} m = \phi^t M \phi \; ; \; a_o = \phi^t A_o \phi \; ; \; a_1 = \phi^t A_1 \phi \; ; \; a_2 = \phi^t A_2 \phi \\ k_o = \phi^t K_o \phi \; ; \; k_1 = \phi^t K_1 \phi \; ; \; k_2 = \phi^t K_2 \phi \; ; \; f = \phi^t F \end{aligned} \tag{10}$$

SOLUTION OF THE EQUATIONS

Instabilities and unbalance response come from the reduced system (9), and from (7). The method used is that detailed in [8]. Here the basic principles are shown.

* Instabilities are found with the resolution of :

$$m q^{\bullet\bullet} + a q^{\bullet} + kq = 0 \quad \text{with :} \tag{11}$$

$$a = a_o + a_1 \sin 2\Omega t + a_2 \cos 2\Omega t \tag{12}$$

$$k = k_o + k_1 \sin 2\Omega t + k_2 \cos 2\Omega t \tag{13}$$

(11) is transformed into :

$$\frac{dp}{dt} = B\ p \tag{14}$$

with

$$p = \begin{vmatrix} q \\ q^{\bullet} \end{vmatrix} \tag{15}$$

and

$$B = \begin{vmatrix} 0 & I \\ -m^{-1}k & -m^{-1}a \end{vmatrix} \tag{16}$$

B is a periodic matrix, of period T. The period T of the periodic coefficients is divided in s intervals of length $h = T/s$. System (14) is considered to be constant on each interval and a matrix T_i connecting displacements and velocities at the instants ih and $(i-1)h$ can be calculated. The general matrix connecting q and $q^{\bullet}$ over a period T is obtained by the product of matrices :

$$T_f = T_{s-1} \ \ldots \ T_j \cdot T_{j-1} \ \ldots \ T_1 \cdot T_o \tag{17}$$

and

$$p(T) = T_f\ p(0) \tag{18}$$

The $2.\ell$ complex eigenvalues of T_f are representative of the stability of the system. If they are all less than unity the system is stable. Here each matrix T_j is calculated with a Newmark formulation and the expression of T_j is given below.

$$T_j = \begin{vmatrix} D^{-1}F & D^{-1}E \\ \frac{2}{h}(D^{-1}F-I) & \frac{2}{h}D^{-1}E - I \end{vmatrix} \tag{19}$$

with

$$D = \frac{4m}{h^2} + \frac{2c_{j+1}}{h} + k_{j+1} \tag{20}$$

$$E = \frac{4m}{h} + c_{j+1} - c_j \tag{21}$$

$$F = \frac{4m}{h^2} + \frac{2c_{j+1}}{h} - k_j \tag{22}$$

Instability zones are obviously the same as those which would be obtained from (1).

* Unbalance response is the solution of (9). Numerical resolution is made with a Newmark formulation and the initial conditions are chosen as :

$$t = 0 \quad q(0) = 0 \tag{23}$$

$$q^{\bullet}(0) = 0 \tag{24}$$

$$q^{\bullet\bullet}(0) = m^{-1} f(0) \tag{25}$$

APPLICATION

A rotor of a 220 KW Steam Compressor, (Fig.1) is studied. The rotor contains symmetric shaft and disks, and dissymmetric bearings. Calculations are at first made in a fixed reference frame where equations have constant coefficients and are obtained with a finite element model and are easily solved [9]. Then, calculations are made in a rotating reference frame : the equations have periodic coefficients and are also obtained with a finite element model. The comparison of the results in the two different frames shows the interest of the method proposed.

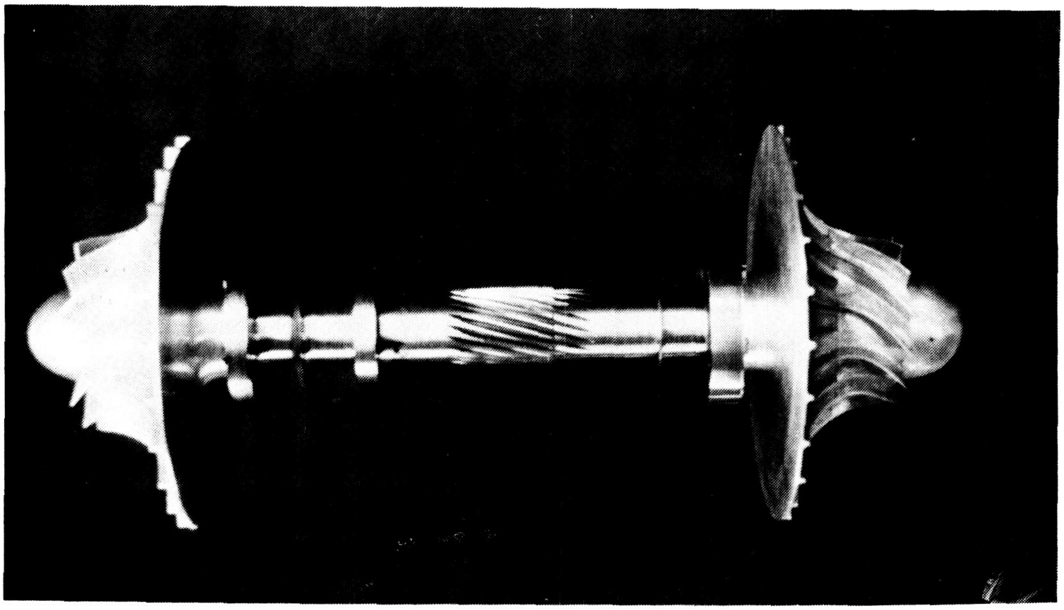

Fig. 1 : 220 KW Steam Compressor Rotor

1 - Description of the model

The rotor is modeled with 23 finite elements, as shown in Fig.2. Two calculations are made with different materials for the disks. They are in Aluminium for rotor A, and in steel for rotor B.

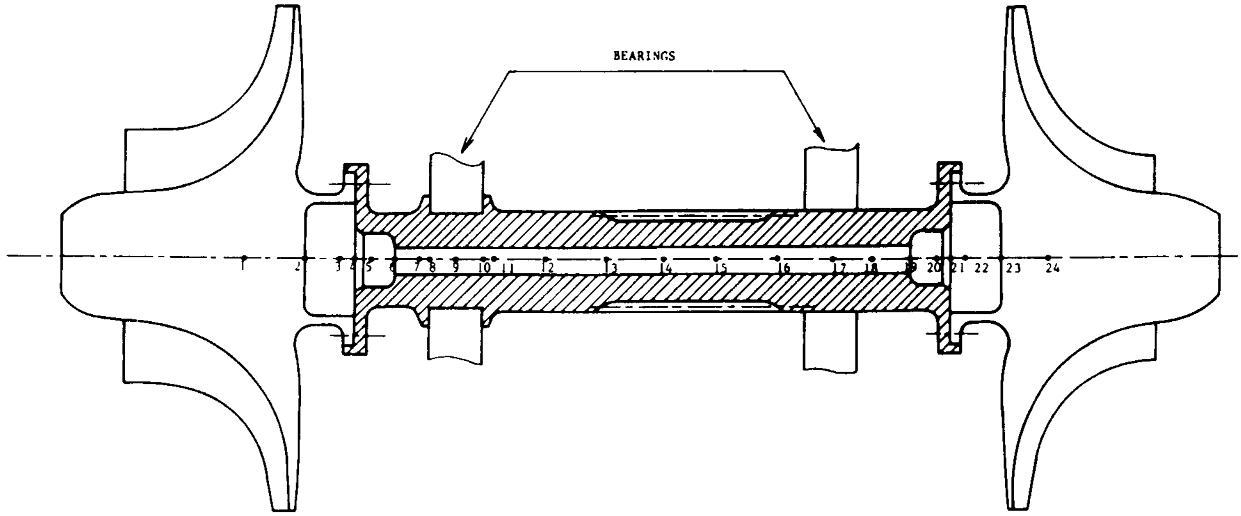

Fig. 2 : Modelisation of the rotor

2 - Calculations

The two rotors are used for different purposes. For rotor A (aluminium disks) the unbalance response can be performed over 4 critical speeds because the motion is always stable in the operating range (0 - 60000 RPM). For rotor B, instabilities appear near 20.000 RPM. So the two configurations will provide satisfactory tests both for instabilities and unbalance response.

3 - Results

* Instabilities

They appear at 21300 RPM in the fixed reference frame. In the rotating one the instability with 14 modes appears at 19700 RPM.

* Unbalance response

Fig. 3, 4, 5 represent the maximum of the unbalance response at node 1. Different number of modes (6-10-14) are considered, and compared to the results in the fixed-reference frame. The resolution in the rotating reference frame with reduced coordinates introduces a slight gap in the frequencies. This creates differences in the amplitude to up to 10 % in the critical frequencies zones. Only the permanent solution is compared, and the number of rotations of the rotor necessary to obtain the permanent solution depends a lot on its rotating speed.

CONCLUSION

The dynamic behavior of periodic coefficient large system is predicted here with a pseudo modal method, using a significant reduction of the number of degrees of freedom.

Differences which are shown in the industrial example presented may be inherent to the numerical calculations.

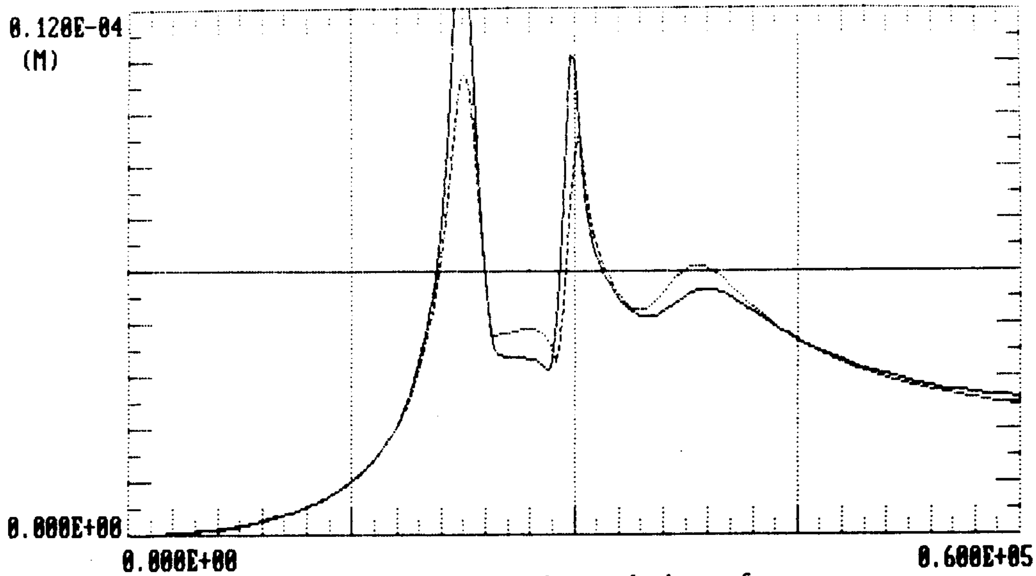

Fig. 3-: 6 modes and the reference

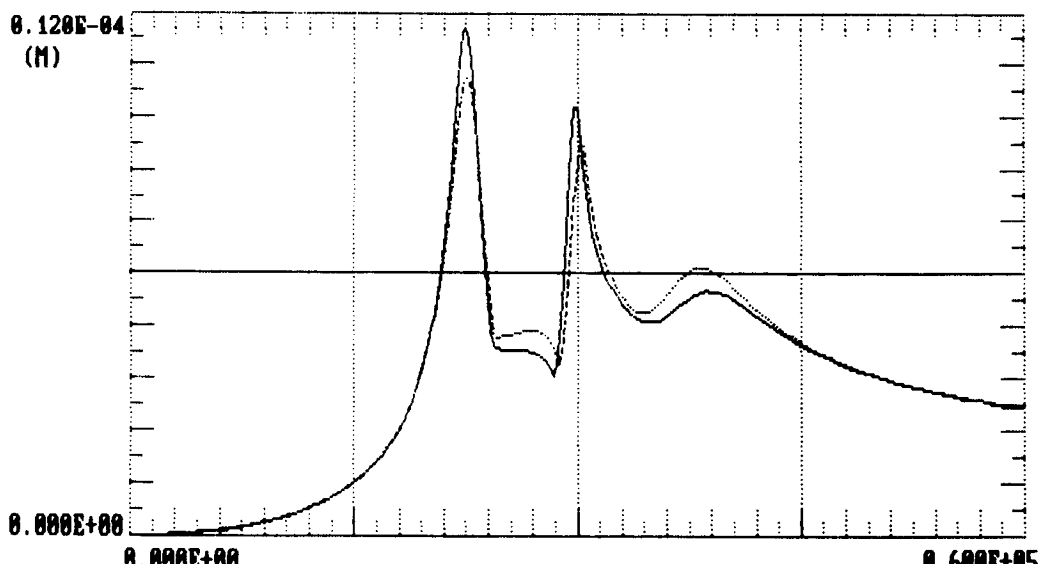

Fig. 4 : 10 modes and the reference

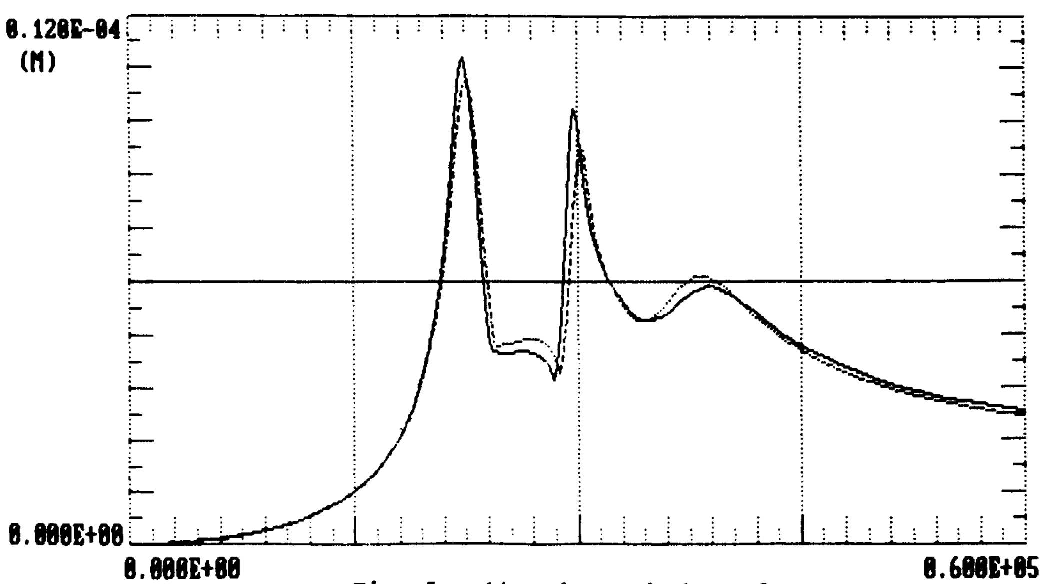

Fig. 5 : 14 modes and the reference

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NOMENCLATURE

A	"Damping" matrix $[n.n]$ containing periodic coefficients
a	modal damping matrix $[\ell.\ell]$
B	matrix which transforms a $[\ell.\ell]$ second order differentiel system into in $[2\ell.2\ell]$ first order differential system
E	Young modulus (N/m^2)
F	unbalance force vector
h	time interval $h = T/s$
I	unity matrix
K	"stiffness" matrix $[n.n]$ containing periodic coefficients
k	modal stiffness matrix $[\ell.\ell]$
ℓ	number of modes taken into acount
M	mass matrix $[n.n]$
m	modal mass matrix $[\ell.\ell]$
n	number of degrees of freedom
P	vector $[2\ell]$ containing modal displacements and vitess
q	modal displacement vector $[\ell]$
s	number of intervals in a period T
T	period of coefficients of differential equations

T_f	transfer matrix over one period T
X	displacements vector $[n]$
b	modal base built with the first ℓ modes
Ω	rotation speed of the rotor
ρ	volumic mass (Kg/m^3)
ν	Poisson coefficient
$\bullet$	d/dt
$\bullet\bullet$	d^2/dt^2